(9) Temos:

- $\Omega = \{(1,1), (1,2), \ldots, (6,6)\}$

  $= A^2, \quad A = \{1,2,3,4,5,6\}$

- $ax - b = 0 \iff x = \frac{b}{a} \in \mathbb{Z} \iff$ b é divisível por a

- $E = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)\}$

- $P(x \in \mathbb{Z}) = \frac{14}{36} = \frac{7}{18}$

---

(10) Se

- $P(A) = 0,3$
- $P(A \cup B) = 0,8$
- $P(B) = p = ?$
- $A \cap B = \emptyset$

então

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

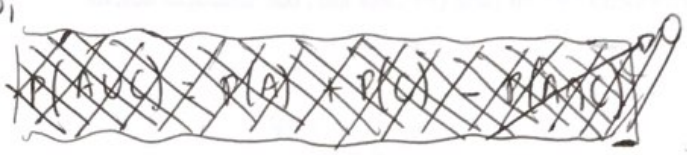
$$0,8 = 0,3 + p - 0$$

$$\boxed{p = 0,5}$$

(11) Temos

- $P(A) = P(B)$
- $P(A) = 2P(C)$

Logo,



- $P(A \cup B \cup C) = 1 = P(A) + P(B) + P(C)$

$$1 = 2P(C) + 2P(C) + P(C)$$

$$\boxed{P(C) = \frac{1}{5}}$$

- $P(A) = 2 \cdot \frac{1}{5} = \frac{2}{5}$

- $P(A \cup C) = P(A) + P(C) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ //

12 · 11 · 10 · ... · 1

(12) $P(\text{mês diferente}) = \frac{12 \cdot 11 \cdot 10 \cdot \ldots \cdot 1}{12^{12}}$

$= \frac{11!}{12^{11}} = 5{,}37 \cdot 10^{-5}$ #

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{25/36}{25/36}$

---

(13)

- 40 moedas
  - 5 moedas → duas caras
  - 10 moedas → normais (cara e coroa)
  - 25 moedas → duas coroas

- $P(\text{coroa e coroa}) = \frac{25}{35} = \frac{5}{7}$

---

(14)

$P(\text{livros juntos}) = \frac{5!\,2!\,3!\,3!}{10!}$

$= \frac{2 \cdot 6 \cdot 6}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} = \frac{1}{60 \cdot 7}$

$= \frac{1}{420}$

(15)

- Bolas numeradas de 1 a 90
- $P(\text{múltiplo de 5 ou 6}) = ?$

$$= P(\text{múlt. de 5}) + P(\text{múlt. de 6}) - P(\text{múlt. de 5 e 6})$$

$$= \frac{\left[\frac{90}{5}\right]}{90} + \frac{\left[\frac{90}{6}\right]}{90} - \frac{\left[\frac{90}{5 \cdot 6}\right]}{90}$$

$$= \frac{18}{90} + \frac{15}{90} - \frac{3}{90}$$

$$= \frac{30}{90} = \boxed{\frac{1}{3}}$$

---

(16)

Temos:

- $|H| = \binom{365}{2} = \frac{365!}{2!\,363!} = \frac{365 \cdot 364}{2}$
- $E = \{\{1,182\}, \{2,181\}, \{3,180\}, \{4,179\}, \ldots, \{83,100\}, \{84,99\}, \{85,98\}, \ldots, \{99,84\}$ (REPETIDO), $\{86,97\}, \{87,96\}, \{88,95\}, \{89,94\}, \{90,93\}, \{91,92\}, \{92,91\}$ (REPETIDO)$\}$
- $P(\text{soma ser } 83) = \frac{91}{\frac{365 \cdot 364}{2}} = \boxed{\frac{1}{730}}$

(17) Temos:

~~$\Omega = \{K, C\}$~~

C: cara e K: coroa $\Rightarrow \Omega = \{$ CCCC, CCCK, CCKC,

CKCC, KCCC, CCKK, CKCK, CKKC, CKKK,

KKKK, KKKC, KCCK, KCKC, KCKK, KKCC,

KKCK $\}$

- $P(\text{pedro ganhar}) = \frac{8}{16} = \frac{1}{2}$
- $P(\text{João ganhar}) = \frac{8}{16} = \frac{1}{2}$

Logo, ambos tem a mesma chance.

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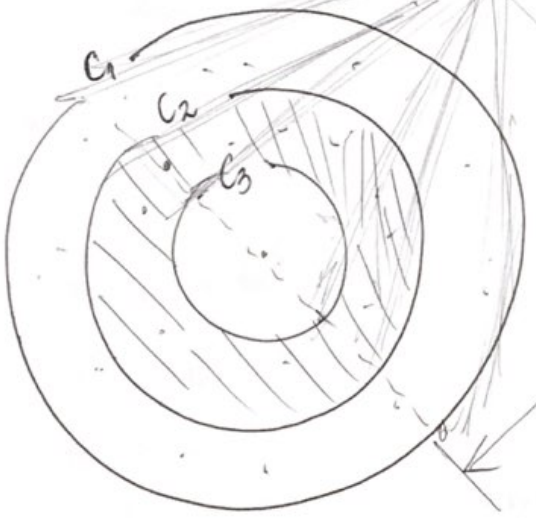
(18)



altura: h

$P(x \in ((C_1 \cap C_2) - C_3)) = ?$

$$= \frac{\frac{1}{3}\pi\left(\frac{R}{2}\right)^2 \cdot h - \frac{1}{3}\pi\left(\frac{R}{4}\right)^2 \cdot h}{\frac{1}{3}\pi \cdot R^2 \cdot h}$$

$$= \frac{1}{4} - \frac{1}{16} = \boxed{\frac{3}{16}}$$

#

(19) Temos:

- 10 postos de gasolina
- 2 vendem gasolina adulterada
- p(dois infratores sorteados) = ?

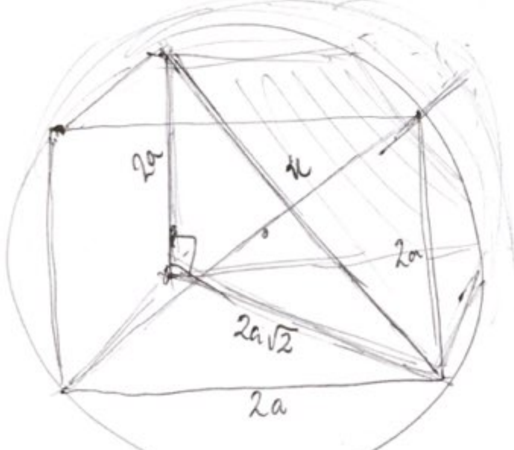
$$\| \qquad \frac{1 \leftarrow \text{um par}}{\binom{10}{2}} = \frac{1}{\frac{10!}{2!\,8!}} = \frac{2}{10 \cdot 9} = \frac{1}{45}$$

de $\binom{10}{2}$ pares

\#

50 [350]

(20)



$x^2 = 8a^2 + 4a^2$

$x^2 = 12a^2$

$x = 2a\sqrt{3}$

$\Downarrow$

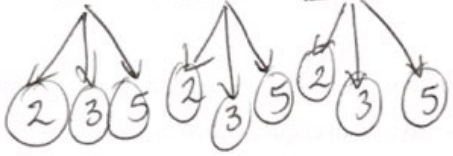
$R_{esf.} = a\sqrt{3}$

$$p\left(x \in (Esf. \cap Cubo)\right) = \frac{(2a)^3}{\frac{4}{3}\pi (a\sqrt{3})^3} = \frac{8a^3}{\frac{4}{3}\pi a^3 \cdot 3\sqrt{3}} = \frac{2}{\pi\sqrt{3}} = \frac{2\sqrt{3}}{3\pi}$$

\#

(21) Temos:

- $\Omega = A^3$, $A = \{1, 2, 3, 4, 5, 6\}$
- $E_1(\underbrace{a, b, c}_{\text{consecutivos}}) = \{(1,2,3), (2,3,4), (3,4,5), (4,5,6)\}$
- $E_2(\underbrace{a, b, c}_{\text{primos}}) = \{(2,2,2), (3,3,3), (5,5,5), (2,2,3), (2,3,2) \ldots\}$
- $|E_2| = \underline{3} \cdot \underline{3} \cdot \underline{3} = 27$



- $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$$= \frac{4}{6^3} + \frac{27}{6^3} = \boxed{\frac{31}{6^3}}$$

(22) Como

- $P(A) = P(B)$
- $P(A) = P(B) = 2P(C)$
- NÃO HÁ EMPATE

então

$$P(A) + P(B) + P(C) = 1 \Rightarrow 5P(C) = 1 \Rightarrow \boxed{P(C) = \frac{1}{5}}$$

Logo,

$$P(B \cup C) = P(B) + P(C) - \cancel{P(B \cap C)}^{\text{zero}}$$

$$= \frac{2}{5} + \frac{1}{5} = \frac{3}{5} = \frac{6}{10} = \boxed{0,6}$$

---

(23) Considere:

E_i : dar cara no lançamento i, $1 \leq i \leq 10$.

Assim,

$$P(E_2 \cup E_4 \cup E_6 \cup E_8) = 1 - P(E_2 \cup E_4 \cup E_6 \cup E_8)^c$$

$$= 1 - P(E_2^c \cap E_4^c \cap E_6^c \cap E_8^c)$$

$$= 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$= \boxed{15/16}$$

$P(E_1 \cup E_5 \cup E_6 \cup \bar{E}_6) = 1 - P(E_1 \cup E_4 \cup E_5 \cup E_8)^C$

$= 1 - P(E_1 \cap E_4 \cap \bar{E}_6 \cap E_8)$

$0 \leq P(A \cup B) \leq 1$

$0 \geq -P(A \cup B) \geq -1$

$1 - \frac{1}{16}$

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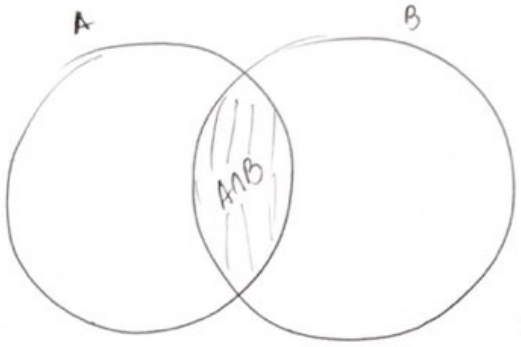
(24)



Como $(A \cap B) \subset B$ então $P(A \cap B) \leq P(B) = \frac{4}{9}$.

Por outro lado, por ser $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

temos:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\geq P(A) + P(B) - 1, \text{ pois } 0 \leq P(A \cup B) \leq 1$$

$$= \frac{2}{3} + \frac{4}{9} - 1$$

$$= \frac{6}{9} + \frac{4}{9} - \frac{9}{9} = \frac{1}{9}$$